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SOME INFORMAL THOUGHTS ON RELATIVITY AND LIMITATIONS ON INTERSTELLAR TRAVEL

Reinald G. Finke

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Limitations on interstellar flight in terms of time, acceleration, energy, and power are evaluated. Certain potential avenues open to propulsion and energy-source technology to achieve acceptable interstellar travel within these limitations are cited. In particular, the capabilities of electron-beam propulsion and antimatter-annihilation energy production are examined.

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I. INTRODUCTION

Relativity is considered by the layman to be an arcane subject, fraught with many popular misconceptions (and some not so popular). Its nature is plagued by an ascending hierarchy of layers of complexity, demanding more and more mathematical agility, and excluding more and more observers, as one penetrates to each subsequent level. (The next level may actually introduce interpretations contrary to those of the previous.) Those nonmathematical observers who would explain it tend to fall into two principal classes: (1) the ones who are enamored of the complexities but who cloak their lack of fundamental understanding in soporific detail of explanation, and (2) the ones who have penetrated to only a shallow depth of complexity and produce oversimplified explanations for the buying public who may have more interest in reading something understandable than something correct. Example products of the two classes can be found readily on public library shelves (see Bibliography).

If mankind is ever to achieve the ultimate goal of traveling to the stars, we will need an *engineering* understanding of relativity (as expressed by "I can get close enough").

This paper returns to the sources, rejecting available overcomplicated and over-simplified explanations, and attempts to generate such an intuitive engineering understanding within the limits of this observer. If it falls short (i.e., into class 2 above) and is contradicted by the next more inaccessible level of complexity, at the very least it throws down the gauntlet to the theoreticians, providing examples of possible misconceptions to be acknowledged and refuted, and illustrating the kind of explanations needed to reach the goal of engineering understanding.

II. BACKGROUND

The Special Theory of Relativity can be stated in terms of the two Einstein Postulates:

1. The laws of nature produce the same physical behavior in all frames of reference moving with uniform velocity, independent of that velocity; i.e., it is impossible to determine the absolute state of motion of an unaccelerated frame by any physical experiment carried out in that frame. (This enunciation is equivalent to the Galilean Principle of Relativity.)

2. The speed of propagation of electromagnetic energy in vacuum, c, has the same magnitude in all directions at all places in all unaccelerated frames, independent of the relative velocity of the frames, i.e., independent of the velocity of a source or a detector.

The Lorentz transformation of coordinates, from one unaccelerated frame in which the four-dimensional location of an event is x, y, z, t, to another parallel frame x', y', z', t', coinciding with the first at t = 0 and moving with a constant velocity v parallel to the x, x' axes with respect to the first, was derived by Einstein (see Einstein, 1920, p. 120, and Bergmann, p. 34) from Postulate 2 from the coexistence of four-dimensional "spherical" electromagnetic wave fronts in the two frames,

$$x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 - c^2t'^2 \quad .$$

This equality gives the Lorentz transformation as (using the notation conventions $\beta = v/c$ and $\gamma = 1/\sqrt{1-v^2/c^2}$)

$$x' = \frac{x - vt}{\sqrt{1-v^2/c^2}} = \gamma(x - \beta ct)$$

$$\left.\begin{array}{l} y' = y \\ z' = z \end{array}\right\} \text{ dimensions perpendicular to the direction of motion do not change}$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}} = \frac{\gamma}{c}(ct - \beta x) \quad .$$

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This transformation can be visualized as a sort of rotation through an angle θ from one hyperbolic (x, ct) coordinate system (notice the minus sign on the c^2t^2 term for the "spherical" wave front) to another (x', ct') described by

$$x' = x \cosh \theta + ct \sinh \theta$$

$$ct' = x \sinh \theta + ct \cosh \theta \quad ,$$

analogous to the familiar polar rotation in two-dimensional Cartesian (circular) coordinates, where $x^2 + y^2$ equals $x'^2 + y'^2$, given by

$$x' = x \cos \theta + y \sin \theta$$

$$y' = - x \sin \theta + y \cos \theta \quad .$$

In the hyperbolic-function version of the Lorentz transformation, the angle θ is defined by

$$\tanh \theta = - v/c = - \beta \quad \text{(from which } \cosh \theta = \gamma) \quad .$$

A principal consequence of Postulate 2 is stated by the time equation of the Lorentz transformation; time measured through transmission of information via electromagnetic radiation appears to progress at different rates at different points in different frames that have differing relative velocities, as quantified by the following:

Consider a device made up of a light source, a light detector, and a mirror in the following relationship (after Einstein):
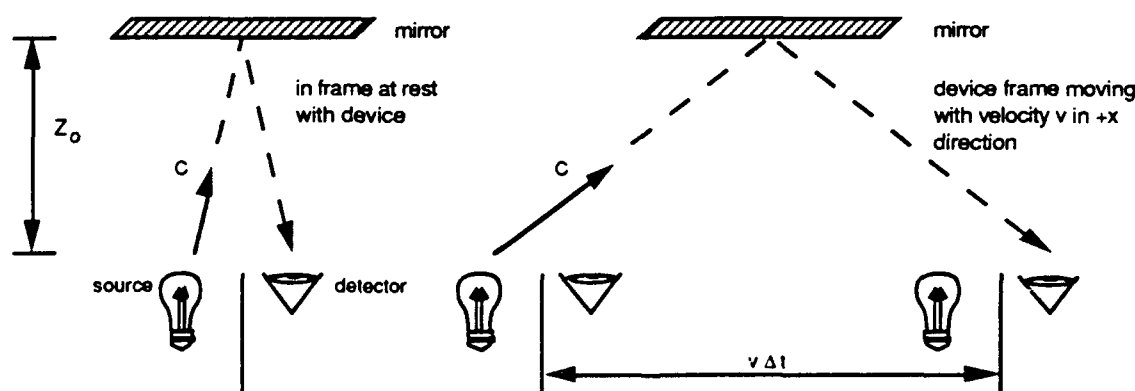
If the source and detector are close enough together so that the value of the cosine of the angle of incidence on the mirror can be taken to be 1, a light pulse emitted by the source in the frame at rest with the device will make a transit to the detector via the mirror in a time $\Delta t_0 = 2z_0/c$. The time interval Δt_0 between two events at a particular location in a frame measured by a clock fixed in the frame at that location is termed the "proper" time interval.

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("Events" may be the departure and arrival of a starship, timed by a clock on its control panel.)

If the frame in which this device is at rest is set in motion along the x axis with a velocity v with respect to an observer (let's call him a "stationary" observer), the "stationary" observer will see the light pulse trace a diagonal path with length $2\sqrt{z_0^2 + (v\Delta t/2)^2}$ at a speed (by Postulate 2) of c. The duration of a transit measured by clocks of a "stationary" observer will be

$$\Delta t = 2\sqrt{z_0^2 + (v\Delta t/2)^2}/c = \sqrt{\Delta t_0^2 + v^2\Delta t^2/c^2} \quad .$$

So the "stationary" observer will measure an elapsed time of transit of a light pulse aboard the frame (starship?) moving with relative velocity v which is greater than for a similar device at rest in his own frame

$$\Delta t = \Delta t_0/\sqrt{1-v^2/c^2} = \Delta t_0/\sqrt{1-\beta^2} = \gamma\Delta t_0 \quad .$$

The time interval Δt between events at a particular point in the moving frame (a starship, say) measured by clocks at different points in the "stationary" frame (stationary with respect to the solar system, say) can be termed an "apparent" time interval. (In this case, the departure and arrival of the starship are timed by different clocks at the departure point and the arrival point; see discussion below for round-trip.) Apparent time is always greater than proper time. The increase in the apparent time measured in the "stationary" frame over the calculated proper time in the moving frame, i.e., $\Delta t = \gamma\Delta t_0$, is referred to as time "dilation" (or "dilatation").

The chief argument used by detractors against special relativity is that an observer in the moving frame (a starship, or Einstein's "relativistic train") would see exactly the same phenomenon when observing the similar device in the "stationary" frame, i.e., the frame moving with respect to his with −v (note: the function of v in the relation between time intervals is v^2); each observer would measure the time interval for a transit in the other frame as greater than in his. Similarly, the Doppler shift of light from epsilon Eridani, say, measured on Sol III (ignoring planetary motion) is *exactly the same* as the Doppler shift of light from Sol measured on epsilon Eridani III. Einstein himself acknowledged this symmetry of such instantaneous observations between equivalent frames, commenting "...this is quite in accordance with the principle of relativity which forms the basis of our considerations" (Einstein, 1920, p. 36). I.e., one cannot determine the absolute state of

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motion of a uniformly translating frame of reference in which one is at rest by any physical measurement carried out in that frame. While such instantaneous measurements may be insufficient for establishing relative time rates, and Einstein's use of the above moving-mirror device to illustrate time dilation may have been pedagogical, there does exist an unequivocal demonstration (among some others achieved since Einstein) of time dilation which was not readily imaginable at the time of Einstein's original paper: a hypothetical round-trip flight of a starship to another star in a frame at rest with respect to the fixed stars (essentially with respect to the Earth).

If the "interval" Δs between two events occurring at different points in four-dimensional space-time which are separated by coordinate differences Δx, Δy, Δz, and Δt is defined by (see the equation above from which the Lorentz transformation was derived)

$$\Delta s = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2} \quad \text{(space-predominant, or "space-like")} ,$$

or by

$$\Delta s = \sqrt{c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2} \quad \text{(time-predominant, or "time-like")} ,$$

this interval is independent of the orientation of the coordinate system ("invariant to a Lorentz transformation" for four dimensions) just as is the conventional distance $\Delta s = \sqrt{\Delta x^2 + \Delta y^2}$ in two-dimensional Cartesian coordinates. If the x and x' axes are chosen to be parallel with the spatial displacement in the stationary frame between the two points at which the events occur, the interval (and its invariance) can be described more simply as

$$\Delta s = \sqrt{c^2 \Delta t^2 - \Delta x^2} = \sqrt{c^2 \Delta t'^2 - \Delta x'^2} .$$

If the two events occur at one point in the moving frame ($\Delta x' = 0$)

$$\Delta s = c \Delta t' ,$$

so $\Delta t'$ is the elapsed time measured by a clock at a particular point and is therefore the elapsed proper time, and Δt is measured by two clocks spaced Δx apart, so is the elapsed apparent time. So, another relation (referred to as "the interval equation" later) between proper time and apparent time is

$$c^2 \Delta t'^2 = c^2 \Delta t^2 - \Delta x^2 .$$

In a round-trip of a starship to a remote point Δx from the origin with return to the origin, the "apparent" duration of the trip Δt can be measured by a single clock fixed at the origin, and is given (from the last equation above) by

$$c^2 \Delta t^2 = c^2 \Delta t'^2 + (2\Delta x)^2 \quad .$$

The difference in the squares of the apparent time and the proper time is

$$\Delta t^2 - \Delta t'^2 = (2\Delta x)^2 / c^2 \quad ,$$

which is a function of distance only, without a dependence on either velocity or the history of the velocity. So, a difference in elapsed-time-squared between the starship's clock and the Earth clock comes about by the starship *traversing space in a frame at rest with respect to a massive object* (the Earth, or the solar system, or the Galaxy).

If the magnitude of the velocity were constant in Δt, then $2\Delta x = v\Delta t$, and the last equation above becomes

$$\Delta t^2 - \Delta t'^2 = (v\Delta t)^2 / c^2$$

$$\Delta t^2 - v^2 \Delta t^2 / c^2 = \Delta t'^2$$

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}}$$

$$= \gamma \Delta t' \quad \text{(constant velocity in } \Delta t\text{)}, \qquad \text{Q.E.D.}$$

Comment on the apparent "dilation" of the lifetime of μ-mesons created from cosmic rays in the upper atmosphere is reserved until later.

The distance (i.e., x) equation of the Lorentz transformation says that distances measured by elapsed times of transmission of electromagnetic radiation appear to be different in frames differing in relative velocity, quantified by the following:

Consider a "ruler" (i.e., gap, distance) with length L_o at rest parallel with the x axis with a mirror attached normal to its far end. A light pulse emitted from its near end will travel at the speed c to the mirror and return to its near end in the proper time interval $\Delta t_o = 2L_o/c$, defining its "proper" length $L_o = 1/2 \, c \, \Delta t_o$.

If the frame in which this ruler is at rest is set in motion along the x axis with a velocity v with respect to the "stationary" observer, the observer will see the light pulse travel at a speed (by Postulate 2) of c from the near end to the far end, which will have been displaced $v\Delta t_1$ by the time the light pulse arrives, in the one-way transit time Δt_1 measured

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in the stationary frame. The apparent distance traversed by the outbound light pulse will be $L + v \Delta t_1$, where L is the apparent length of the ruler, to be determined. Equating this apparent distance to the distance traveled by light in Δt_1 gives the implicit relation

$$c \Delta t_1 = L + v \Delta t_1 \quad .$$

Solving explicitly for Δt_1 gives

$$c \Delta t_1 = L/(1 - \beta) \quad .$$

Similarly, the return trip takes place in the transit time Δt_2 measured in the stationary frame, determined by

$$c \Delta t_2 = L - v \Delta t_2 \quad .$$

Solving explicitly for Δt_2 gives

$$c \Delta t_2 = L/(1 + \beta) \quad .$$

So the apparent round-trip time of the light pulse is given by (using $\gamma = 1/\sqrt{1-\beta^2}$)

$$
\begin{aligned}
c(\Delta t_1 + \Delta t_2) &= L[1/(1-\beta) + 1/(1+\beta)] \\
&= L[(1+\beta)/(1-\beta^2) + (1-\beta)/(1-\beta^2)] \\
&= 2L/(1-\beta^2) \\
&= 2\gamma^2 L \quad ,
\end{aligned}
$$

so the apparent length is

$$L = c(\Delta t_1 + \Delta t_2)/(2\gamma^2) \quad .$$

But from the analysis preceding this, the apparent time interval $(\Delta t_1 + \Delta t_2)$ in the stationary frame is greater than the proper time interval Δt_o in the moving frame by the factor γ, or

$$\Delta t_1 + \Delta t_2 = \gamma \Delta t_o \quad .$$

Substituting $\gamma \Delta t_o$ for $(\Delta t_1 + \Delta t_2)$ in the expression for L above and replacing Δt_o with $2L_o/c$ gives

$$L = c\gamma \Delta t_o/2\gamma^2 = c\gamma(2L_o/c)/2\gamma^2 \quad .$$

So the *apparent* length of the ruler is

$$L = L_o/\gamma \quad ,$$

i.e., shorter than the *proper* length by the factor $1/\gamma$ ("Lorentz-FitzGerald contraction").

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The time-dilation and FitzGerald-contraction relations can be obtained directly from the Lorentz transformations (Schwartz, p. 41) between coordinates x and t in a stationary frame and coordinates x' and t' in a frame moving at a velocity v in the plus-x direction.

The *apparent* difference in time (Δt) in the rest frame of a time interval ($\Delta t'$, or *proper* time) between two events occurring at the same point in the moving frame ($\Delta x' = 0$) is determined from the Lorentz-transformation equations

$$\Delta x' = \gamma(\Delta x - \beta c \Delta t) \quad , \text{ and}$$

$$c \Delta t' = \gamma(c \Delta t - \beta \Delta x) \quad .$$

For $\Delta x' = 0$, the first equation becomes

$$\Delta x = \beta c \Delta t \quad .$$

Substituting this in the second equation leads directly to the time-dilation relation

$$\Delta t' = \gamma(\Delta t - \beta^2 \Delta t) = \gamma \Delta t (1 - \beta^2) = \gamma \Delta t / \gamma^2 \quad , \text{ or}$$

$$\Delta t = \gamma \Delta t'$$

(apparent time is greater than proper time by the factor γ).

The *apparent* length $\Delta x'$ in the moving frame of a ruler at rest with a *proper* length Δx is given by the simultaneous measurement ($\Delta t' = 0$) of the positions of the end points, in terms of the reverse-Lorentz-transformation x equation

$$\Delta x = \gamma(\Delta x' + \beta c \Delta t') \quad .$$

For $\Delta t' = 0$, the FitzGerald-contraction equation is obtained directly, i.e., as

$$\Delta x' = \Delta x / \gamma$$

(apparent length is shorter than proper length by the factor $1/\gamma$).

Retaining the Lorentz-transformation notation that uses unprimed variables for coordinates in the stationary frame and primed variables for coordinates in the moving frame, we have the velocity v, the motion of the moving frame, given by dx/dt, i.e., a *proper* distance divided by an *apparent* time. The inverse "motion" of the stationary frame, $-v$ with respect to the moving frame, is given by dx'/dt', i.e., an *apparent* distance divided by a *proper* time. In an example of a starship traveling at a constant speed from Sol to alpha Centauri, say, the apparent velocity v represents either the quotient of the distance between the two stars measured in the frame at rest with their center of gravity (essentially the "Earth" frame) and the dilated ship's elapsed time measured by an "Earth" observer, or

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the quotient of the contracted distance between the stars measured by a starship observer and the elapsed time read on the starship's clock.

But what might be the significance of a velocity u (that equals γv) which is composed of a *proper* distance divided by a *proper* time? This "proper" velocity would represent the quotient of the distance between the stars measured in the "Earth" frame, which distance the starship observer can find listed in his star catalog, and the elapsed time measured by the starship's clock; this would be the effective speed as far as a starship passenger would be concerned. The proper velocity u happens to be the spatial component of the 4-vector invoked in relativity theory to provide a velocity that is invariant to a Lorentz transformation; it becomes also the velocity term of the three-dimensional relativistic momentum

$$p = mu = m\gamma v \quad ,$$

that retains adherence to the fundamental relation between force F and rate of change of momentum

$$F = dp/dt$$

(where the F and t are measured in the same frame of reference).

From the fourth (i.e., time) component of the 4-vector u ($u_4 = ic\gamma$) and from the differential of the magnitude of the proper-velocity 4-vector u with respect to proper time (following Ugarov, p. 143)

$$\frac{d(u^2)}{dt'} = \frac{d(u_1^2)}{dt'} + \frac{d(u_2^2)}{dt'} + \frac{d(u_3^2)}{dt'} + \frac{d(u_4^2)}{dt'} = 0 \quad ,$$

using $F = dp/dt$ and noting the similarity of the results to the classical relation between rate of change of kinetic energy and the scalar product of the force and velocity 3-vectors, which is

$$d(1/2 \, mv^2)/dt = F \cdot v \quad ,$$

one can derive the expression (as did Einstein)

$$E = m\gamma c^2$$

for the total energy of a moving body and, for zero velocity ($\gamma = 1$), the rest energy

$$E = mc^2 \quad .$$

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No physical significance is attributed to the velocity u in the theoretical analysis, and the attention of the relativists immediately returns to the expression γv. In some empirical situations, experimenters have even chosen (erroneously, it turns out) to lump the γ with the rest mass m in the relativistic momentum expression to create a "mass-amplification" concept; i.e., accelerators become "ponderators" (Halliday). In the limited cases in which the force is directed normal to the velocity vector, this concept allows use of familiar laws of acceleration with m replaced by $m\gamma$.

The rest mass m is defined as the ratio of force to acceleration. In a general relation in which the velocity of the moving frame is not parallel with the x-axis, the Lorentz transformation (dealing with the apparent velocity v, of course) produces a force vector that is not parallel with the acceleration vector, so mass (the liaison $m\gamma$) becomes a "tensor," i.e., an operator that converts one vector into another, rather than a "scalar," i.e., a pure quantity that has no inherent direction. On the other hand, the liaison γv has its own problems: in equations of motion (expressed in terms of the apparent velocity v) where the force is applied in a general direction with respect to velocity, the time derivative of γv introduces unfamiliar complicated terms because γ is a function of v. Significant simplifications in formulation and understanding of relativistic equations of motion (cf. Krause) would occur if the integration could be carried out in the proper-velocity domain and the solutions transformed back to the domain of apparent velocity, if that velocity is really desired.

Some useful relations between u and v are the following If the quantity n is defined as the ratio u/c, then

$$v = \frac{n}{\sqrt{n^2 + 1}} \, c = \frac{u}{\sqrt{1 + u^2/c^2}}$$

whence

$$n = \beta\gamma = \sqrt{\gamma^2 - 1} \quad .$$

In the hyperbolic-function version of the Lorentz transformation, where (ignoring minus signs)

$$v = c \tanh \theta \qquad (\tanh \theta = \beta) \quad ,$$

the proper velocity is given by

$$u = c \sinh \theta \qquad (\sinh \theta = \beta\gamma = n) \quad .$$

Note that n is greater than 1 for all β greater than $\sqrt{2}/2 \approx 0.707$.

If a particle with a charge e is accelerated through a voltage V, its kinetic energy is eV, to be equated to the difference between total and rest energy from above

$$KE = eV = m\gamma c^2 - mc^2 = (\gamma - 1)\ mc^2 \quad,$$

so the energy factor γ is

$$\gamma = (KE/mc^2) + 1 = (eV/mc^2) + 1 \quad.$$

Setting

$$\gamma^2 = n^2 + 1 = (KE/mc^2 + 1)^2$$

$$= (KE/mc^2)^2 + 2(KE/mc^2) + 1 \quad,$$

gives

$$n = (KE/mc^2) \sqrt{1 + (2mc^2/KE)}$$

$$= (eV/mc^2) \sqrt{1 + (2mc^2/eV)} \quad.$$

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III. PROPULSION IMPLICATIONS

For electrons ($mc^2 \approx 0.51$ MeV) from the 50-GeV Stanford electron linear accelerator (SLAC), for example, the value of $\gamma - 1$ is about 98,000. The value of n is essentially the same as γ (smaller by a difference $1/(2\gamma)$, or by only about 5×10^{-6}). The momentum of an electron exiting the accelerator, mu (= mnc), is therefore about 98,000 times mc. The energy carried away by that electron, $m\gamma c^2$, is also large and represents a substantial equivalent mass depletion, $m\gamma$ (including the electron mass), from the accelerator/power supply. If this accelerator were viewed as a thruster,[*] the thrust would be given by the momentum of an exhaust electron times the number of electrons exiting per second, but the specific impulse (the thrust per unit mass flow rate) would be given by the momentum per exhaust electron divided by the accompanying depletion in mass of the accelerator/power supply, i.e., by γ electron masses, or

$$I_{sp} = mnc/(m\gamma g_c) = 3.057 \times 10^7 \ (n/\gamma)^{[**]} \ \text{kgforce-sec/kgmass} .$$

In more tractable units, using simply "sec" to represent "kgforce-sec/kgmass" and making use of one year (365.25 days, i.e., the "year"[***] in "light-year") equals 3.156×10^7 sec, the specific impulse for an exhaust velocity c (i.e., $n/\gamma = 1$) is very closely equal to one *year* (0.9686 yr). The I_{sp} for the 50-GeV SLAC beam, with a value of $n/\gamma \ [\approx 1 - 1/(2\gamma^2)]$ differing from 1 by $\sim 5 \times 10^{-11}$, is therefore essentially one year. (The electron accelerator energy could be reduced to a little over 3 MeV without decreasing the I_{sp} by more than 1 percent from its maximum value.) If dumping of one proton (at negligible speed) per electron were required for charge neutralization, the I_{sp} would be reduced by a factor of $\gamma/(\gamma + 1837)$ (where 1837 is the ratio of the proton mass to the electron mass). If a beam of

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[*] The author is unaware of any previously published discussion of such an electron-beam propulsion system.

[**] Even though n/γ can be replaced with β, the ratio is continued to be used because the author feels it conveys more physical insight of both the proper-velocity basis of the derivation and the velocity/mass dependence of I_{sp}.

[***] Perhaps some day distances to stars will be so well determined that a distinction will be necessary between the sidereal year (the time for one revolution of the Earth around the Sun with respect to the fixed stars: 31,558,149.54 sec) and the tropical year (the time between successive passages of the Sun through the vernal equinox: 31,556,925.98 sec).

slow positrons could be used instead of protons, the degradation in I_{sp} would be only by a factor of $\gamma/(\gamma + 1)$; for positrons accelerated to the same energy as the electrons, the degradation would be nil.

Even though the exhaust velocity of a thruster composed of a relativistic electron accelerator is the proper velocity of an electron, u ($= nc$), the accompanying mass depletion by a factor γ times the electron mass reduces the I_{sp} to $nc/(\gamma g_c)$. If the instantaneous proper velocity of a ship (with mass M) driven by this thruster is U, the instantaneous momentum transfer in the frame of the ship is

$$MdU/dt' = -g_c I_{sp} \, dM/dt'$$

$$= -(nc/\gamma) \, dM/dt' \quad .$$

Integrating this momentum-balance relation gives the relativistic rocket equation in a familiar form, in terms of proper velocities now rather than apparent velocities (where M_o is the ship's initial mass and M_f its final mass)

$$\Delta U = g_c I_{sp} \ln (M_o/M_f)$$

$$= (nc/\gamma) \ln (M_o/M_f)$$

$$= (nc/\gamma) \ln (1+M_P/M_f) \quad ,$$

or the required propulsive mass $M_P = M_o - M_f$ is

$$M_P = M_f \left[\exp (\gamma \Delta U/nc) - 1\right] \quad .$$

While this relation addresses a mass depletion M_P that is required to give a value of ΔU (the effective increment in velocity of the starship) of c or more, it does not include considerations for time, power, or energy for traversing interstellar distances.

Acceleration at one g (measured by an inertial platform aboard the ship) for 0.9686 year (measured by the ship's clock) is required to produce a ΔU of c (determined by an integrating accelerometer aboard the ship). Proper velocity is given by the product of proper acceleration and proper time.

The traversed proper distance, calculated by the ship's computer, would be 0.4843 light-years to achieve c at one g. One g can be rephrased as being an acceleration of essentially one (1.0324) light-year per year per year. Proper distance is one-half the product of proper acceleration and the square of proper time. If an interstellar trip is to be made to a point s light-years (proper distance) from the Earth, at a proper acceleration of

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one light-year per year per year (0.9686 g) half way with a deceleration of the same magnitude for the second half, the elapsed proper (starship) time for the one-way trip is

$$t'_s = 2\sqrt{s} \quad \text{years.}$$

The elapsed apparent (Earth) time, i.e., half the round-trip time, is determined by the interval equation as

$$t_s = \sqrt{t'^2_s + s^2} \qquad (c = 1 \text{ light-year/year})$$
$$= \sqrt{4s + s^2} \quad \text{years} .$$

So for such a trip to a point 10 light-years away, say, the one-way time on the starship's clock would be 6.32 years ($\bar{u}$ = 1.58 c), while the time on an Earth clock would be 11.83 years ($\bar{v}$ = 0.85 c). The extreme radius in interstellar space that could be reached via such a 0.9686-g acceleration/deceleration profile in a "lifetime" round-trip of 40 years, say, is 100 light-years, with an Earth elapsed time for the round trip of 204 years.[*]

In conventional nonrelativistic terms the "jet" power P for an exhaust-mass rate $\dot{m}$ and an exhaust velocity u is given by

$$P = 1/2 \, \dot{m}u^2 \quad ,$$

but the thrust T is

$$T = \dot{m}u \quad ,$$

so the nonrelativistic P is

$$P = 1/2 \, Tu \quad .$$

Relativistically the thrust equation still holds, with u equal to the proper velocity of an exhaust particle and $\dot{m}$ equal to the mass of the particle times the number exhausted per second

$$\dot{m} = m\dot{i}/e$$

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[*] Cf. analysis cited in Taylor and Wheeler, p. 97, and Schwartz, p. 122 (which does not conform with the interval equation), and, for the 40-year-proper-time round trip, resulting values of over 20,000 light-years for radius and over 40,000 years for Earth elapsed time, stated in Bondi, p. 153, and Taylor and Wheeler, loc. cit.

where m is the particle mass, e is the charge of the particle, and i is the beam current of the accelerator/thruster. So the thrust equation becomes*

$$T = \dot{m}u = (mi/e) \, nc$$

$$= (mi/e) \, c \, (eV/mc^2) \sqrt{1+2mc^2/eV)}$$

$$= (iV/c) \sqrt{1+2mc^2/eV}$$

$$= (P/c)\sqrt{1+2mc^2/eV}$$

$$\approx P/c............if \ eV >> 2mc^2 \quad .$$

Note also that, for $eV >> 2mc^2$, the thrust is *independent of the particle mass*; there is no penalty in using the more easily accelerated electrons rather than ions.

If u is small so that $eV = 1/2 \, mu^2 << 2mc^2$

$$T \approx (P/c) \sqrt{2mc^2/(1/2 \, mu^2)}$$

$$= 2 \, P/u........the nonrelativistic relation, Q.E.D.$$

If the beam power is supplied by conversion of the energy of antimatter/matter annihilation (forget fusion; it takes at least 157 times the mass to produce the same energy) at a rate $\dot{M}_a$ by the ship's clock with a conversion efficiency η, then

$$P = \eta \dot{M}_a c^2 \approx Tc \quad .$$

The acceleration of the ship measured by an on-board accelerometer is given by

$$MdU/dt' = T \approx P/c = \eta \dot{M}_a c \quad .$$

If the rate of diminution of the ship's mass dM/dt' becomes predominantly $\dot{M}_a$ (to be verified below), this equation can be written

$$MdU/dt' = - \eta c dM/dt' \quad , \ or$$

$$dU = - \eta c dM/M \quad .$$

Integrating this equation gives an "energy" rocket equation in terms of the total annihilation mass M_a (which is twice the antimatter mass)

$$\Delta U = \eta c \ln (1 + M_a/M_f) \quad ,$$

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* More directly, from $pc \approx E$ for highly relativistic particles (or $pc = E$ for photons, for a "photon rocket"), T = rate of ejection of momentum $\approx$ (rate of ejection of energy)/c = P/c.

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and the required annihilation mass is given by

$$M_a = M_f [\exp(\Delta U / \eta c) - 1] \quad .$$

This equation is in exactly the same form as the rocket equation derived previously, with η (a value less than 1) replacing r/γ (a value that could be very close to 1). Hence, as assumed, the "propulsive" mass will be less than the annihilation mass, and the *energy* rocket equation becomes the operative one in defining the "fuel" requirement for a ship to achieve proper velocities comparable to c. (Note that if η were 0.5 and a ΔU of c were desired, a mass ratio of e^2 would be required; about 43 percent of the initial mass of a single-stage ship should be antimatter.)

If the voltage of the electron accelerator were 100 gigavolts ($\gamma \approx 200{,}000$), say, the beam current required to produce an acceleration of one g would be only 0.03 amps per kg of ship mass, but the beam power would be about 3,000 (actually, 2,940) megawatts per kg of ship mass (to be compared with about 15,000 megawatts of jet power from the Space Shuttle's three main engines, but only about 0.13 megawatts per kg of the Orbiter at burnout). Recall that, for highly relativistic beams, the required power depends only on the desired thrust, and the required beam current varies inversely with the beam voltage. The choice of voltage should be such that the weight of the accelerator is a minimum while keeping the beam power constant at the desired value.

The specific energy of annihilation is about 9×10^{10} megawatt-sec per kg of matter/antimatter, so the power required for a 1-g acceleration represents an antimatter consumption rate (half the annihilation rate) of only $16/\eta$ micrograms/sec per kg of ship mass. Note incidentally that the equivalent explosive energy in the "tanked" antimatter is 47.3 megatons of TNT per kg.

(In a more mundane context, let us consider an application of such a propulsion system to an Earth-Mars one-way trip of, say, 300 million km. At 1-g acceleration half way followed by 1-g deceleration the other half, the trip time would be 4.05 days. The peak speed achieved half way would be 1,715 km/sec. The required amount of antimatter "fuel" would be $5.7/\eta$ gm per kg of ship mass.)

The interstellar propulsion system must overcome the drag caused by encounters with particles (chiefly hydrogen atoms, i.e., protons) in the interstellar "void"; the starship occupants must be shielded from the ionization caused by the interaction of the penetrating particles with matter; and the shield must be able to absorb and reject the resulting heating.

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The drag D per cm^2 of a starship with proper velocity u (= nc) encountering ρ_n protons (with individual mass m) per cm^3 is (equivalent to drag from free-molecule flow with a drag coefficient of 2)

$$D = \rho_n mu^2 = \rho_n mc^2 n^2$$

$$= 1.50 \times 10^{-3} n^2 \rho_n \text{ dynes/cm}^2 \quad .$$

For an extremely lightweight ship with a mass per unit area of only 1 gm/cm^2 (about 2 lb/ft^2) moving at n = 2 through a "thick" void with 10 $protons/cm^3$, say, this drag would impart a deceleration of only 6×10^{-5} g, negligible with respect to the 1 g required of the propulsion system to build up the ship's proper velocity to c in a year.

The kinetic energy KE of a proton with respect to the starship moving at a proper velocity of nc is

$$KE = mc^2 (\gamma - 1) = mc^2 (\sqrt{n^2+1} - 1)$$

$$= 938 \text{ MeV } (\sqrt{n^2+1} - 1) \quad .$$

The shielding thickness, i.e., stopping distance or "range" R, of aluminum for protons with kinetic energy KE is approximately given by[*]

$$R = 3.566 \text{ cm Al } [KE \text{ (MeV)}/100]^{1.76} \quad .$$

If the ρ_n $protons/cm^3$ incident at a proper velocity of nc are stopped by the shielding, they will deposit all their incident kinetic energy in the shielding with an energy flux, i.e., heating rate H per cm^2, of

$$H = \rho_n ncKE \quad .$$

If this heating rate is rejected by reradiation from the surface of the shield with an emissivity of 1, the heating rate can be expressed as an equivalent radiative-equilibrium surface temperature T_{eq}, given by (with σ the Stefan Boltzmann constant)

$$T_{eq} = (H/\sigma)^{1/4}$$

$$= 944K \left[\rho_n n(\sqrt{n^2+1} - 1)\right]^{1/4} \quad .$$

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[*] From data in Rich and Madey, UCRL-2301, 1954.

The resulting values for proton kinetic energy KE, proton range R, and equivalent radiative-equilibrium surface temperature T_{eq} (with the last for both 1 and 10 protons/cm^3) are given as a function of the proper-velocity ratio n in the following table.

n	KE (MeV)	R (cmAl)	T_{eq}, K (1 p/cm^3)	T_{eq}, K (10 p/cm^3)
0.5	111	4	465	827
1.0	389	39	757	1,347
1.5	753	125	989	1,759
2.0	1,159	266	1,184	2,105

The proton kinetic energies are within the experience of particle accelerators, the shielding thicknesses for protons of those energies are equivalent in mass to a few meters of aluminum, and the temperatures to reject the proton heating, in the range of proton densities expected in the interstellar medium, are within known material (other than aluminum) limits.

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IV. CRITIQUE

The integration above of the ship's momentum balance to derive the rocket equation involves the summation of a sequence of proper-velocity increments. The conventional practice is to sum a succession of Lorentz transformations for a sequence of apparent-velocity increments. This latter procedure in effect involves summation of incremental velocity parameters, where the velocity parameter is the argument θ of the hyperbolic functions in the Lorentz transformation. These two procedures must necessarily give the same values for final proper velocity, since

in proper-velocity integration,

$$u_f = \int_0^{u_f} du = \int_0^{\sinh \theta_f} d(\sinh \theta) = \sinh \theta_f \quad ,$$

and in velocity-parameter integration,

$$u_f = \sinh \left(\int_0^{\theta_f} d\theta \right) = \sinh \theta_f \quad .$$

The increased difficulty in accelerating relativistic particles in high-energy particle accelerators has been attributed (e.g., by Halliday) to an increase in their mass ($m\gamma$) as β approaches 1. The slowing down of the frequency of the accelerating electric field in a frequency-modulated cyclotron, or synchrotron, to match the lower-than-expected (assuming constant mass of the particle) acceleration, and the increasing inability of electron linear accelerators to accelerate electrons as their apparent speed approaches the phase velocity of the rf energy, have been ascribed to an increase in mass of the particles. While the accelerating electromagnetic-field energy is radiated from the electrodes, or coupling loops, at the speed c, it must be remembered that it also arrives at the accelerated particle with a relative speed c, no matter what the speed of the particle, and the energy of the arriving photon is Doppler-shifted toward zero as the particle's apparent velocity approaches c. The particle/field interaction needs to be reexamined from this fundamental basis rather than following the conventional "mass-amplification" approach.

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Using the argument that a rocket ship cannot be accelerated to c because its mass goes to infinity at that speed, one can reason that the propellant mass goes to infinity in exactly the same way; the dM/M in the rocket equation does not change, and the acceleration sensed by an accelerometer aboard ship does not diminish as the proper velocity approaches and exceeds c.

A key example cited in corroboration of the existence of time dilation is the apparent extension of the lifetime of relativistic μ-mesons created in collisions between primary cosmic rays and nuclei in the upper atmosphere. The laboratory-measured 2-microsecond lifetime of the μ-meson would allow it to travel only 600 meters at a speed c before it decayed, yet the μ-meson is observed at ground level 6 km below its altitude of origin. It is reasoned therefore that the rate of intrinsic ("proper") time aboard the μ-meson must be slowed down by a factor of 10. The same result could be obtained, however, by viewing the meson's "proper" velocity to be 10 times c.

V. CONCLUSIONS

Accepting the startling deduction that the speed in vacuum of electromagnetic radiation with respect to any source or detector is the same (c), no matter what the velocities of the source or detector may be, as the great breakthrough of the Special Theory of Relativity, one can reason that, no matter how great the proper velocity of a material object may be with respect to an observer, the "stationary" observer will measure an apparent velocity less than c using electromagnetic radiation as the medium of information transmission.

One can reason further that, using "proper" state values (i.e., values measured in the frame of reference at rest with respect to the measurement), the Newtonian laws of mechanics (without "mass amplification") can be used to give a satisfactory, "engineering" description of motion for interstellar travel.

While the reasoning here removes the speed limit from interstellar travel (for the passenger), considerations of acceleration limits and energy requirements indicate that transit times of interstellar distances will still be the order of years, and the energy requirement will be many orders of magnitude above the present supply, to be satisfied only by antimatter annihilation. The propulsion system is envisioned to be a high-voltage, high-current relativistic electron accelerator with a beam power of about 3000 megawatts per g of acceleration per kg of ship mass. Interstellar travel in a passenger's lifetime will require that a significant fraction of a starship's initial mass be composed of antimatter.

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